FILM CONDENSATION OF VAPOR ON HORIZONTAL CORRUGATED TUBES

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The film condensation of vapor on a horizontal tube with trapezoidal fins is calculated analytically for the case where surface tension is predominant. Experimental results for water and Freon-113 vapors are also shown.

The article deals with an analytical and an experimental study of heat transfer during film condensation of a stationary vapor on horizontal tubes with circumferential trapezoidally shaped fins (Fig. 1a, b), under conditions where the effect of surface tension is predominant. As has been mentioned in [7, 8], it is possible with such a finning to considerably enhance – as compared with smooth tubes – the process of heat transfer from vapor to tube wall.

Such a finning would probably be most effective on the surface of a horizontal tube, where surface tension will constrict the liquid condensing on the fins – around the entire tube circumference – into the drains (recesses between fins). At the same time, the fin crests (overhangs) become cleared of liquid and vapor condensation will occur primarily there. One can realize such a pattern of condensate film flow by properly designing the dimensions of the basic fin elements so that the surface tension at the crest surface will be greater than the force of gravity by at least one order of magnitude, i.e., so that We ≥ 10.

The Weber number, which is the ratio of these two forces, can be expressed as follows:

$$\mathrm{We} = \frac{\partial p/\partial x}{\rho} \geqslant \frac{\sigma \cos\varphi}{b(1+\operatorname{tg}\varphi)h\rho}, \tag{1}$$

where the pressure gradient $\partial p/\partial x$ at a certain depth of fin immersion Δ (Fig. 1) may be expressed as

$$\left|\frac{\partial p}{\partial x}\right| \approx \frac{\Delta p}{\Delta x} \approx \frac{\sigma \cos\varphi}{R_{\text{т}}(h-\Delta)}, \tag{2}$$

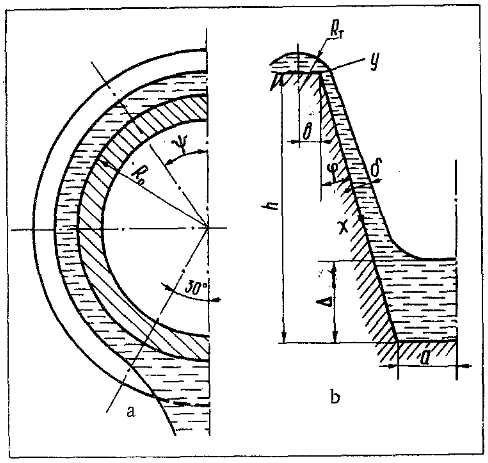

Fig. 1. Model representing the film condensation of vapor on a trapezoidal fin, when We ≥ 10.

and the radius of the liquid film curvature R_T at the top of a trapezoidal fin can be determined, to a first approximation, by the formula:

$$R_{\text{т}} = b(1+\operatorname{tg}\varphi). \tag{3}$$

It is evident from (1) that by decreasing b and angle φ, with given physical properties (σ, ρ) of the substance, one can always attain the condition at which We ≥ 10.

While observing the basic requirement We ≥ 10, we will analyze the motion of a condensate film along a horizontal tube with circumferential fins under the following assumptions:

1. The thin condensate film on the overhang will be treated as a laminar boundary layer, with the pressure gradient along the profile determined by the prevalent surface tension. The effect of gravity and

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forces on the motion of the film along the lateral surface of a fin toward the bottom will be disregarded.

2. The motion of the condensate into the drain is laminar and occurs due to gravity. Vapor condensation directly at the bottom is not taken into account, because the film is very thick here.

3. The wall temperature will be assumed constant across the height of the fin.

Starting out with the basic assumptions concerning the mechanism of heat transfer through a liquid film, which were introduced by Nusselt [6] in an analysis of vapor condensation on smooth surfaces, and solving the differential equation of condensate flow along the fin into the drain (Fig. 1b)

$$-\frac{1}{\mu}\frac{\partial p}{\partial x}+\frac{\partial^2 W_y}{\partial y^2}=0 \tag{4}$$

with the boundary conditions $W_y = 0$ at $y = 0$ and $\partial W_y/\partial y = 0$ at $y = \delta$, we find the mean film velocity over the fin height

$$\tilde{W}_y=-\frac{\delta^2}{3\mu}\cdot\frac{\partial p}{\partial x}=\frac{\sigma\delta^2\cos\varphi}{3(h-\Delta)(1+\operatorname{tg}\varphi)b\mu} \tag{5}$$

and the thickness of the condensate film on the fin

$$\delta=\left(\frac{4\mu\lambda(t_s-t_W)(1+\operatorname{tg}\varphi)(h-\Delta)bx}{\rho r\sigma\cos\varphi}\right)^{1/4} \tag{6}$$

as functions of the physical properties of the condensing liquid, of the fin dimensions, and of the yet unknown liquid height.

This part of the analysis has been presented more completely in [3].

In order to calculate the motion of a liquid layer Δ along a trapezoidal channel between fins under the action of gravity, we need some additional assumptions:

1. The motion of the liquid is laminar with respect to the lateral fin surface immersed in the layer Δ. The effect of the tube wall section (base a) is disregarded.

2. Within the immersion zone, along the y-coordinate from the fin wall to the groove axis the velocity distribution is considered semiparabolic; according to Nusselt, for a two-dimensional flow of a liquid film under the action gravity:

$$U_y=\frac{\rho\sin\psi}{\mu}\left(\delta_{\text{det}}y-\frac{y^2}{2}\right), \tag{7}$$

where

$$\delta_{\text{det}}=(a+\Delta\operatorname{tg}\varphi).$$

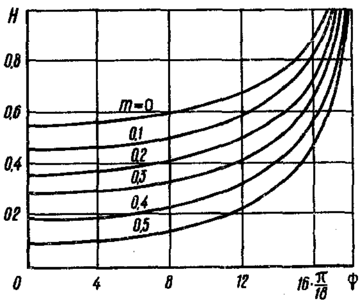

Fig. 2

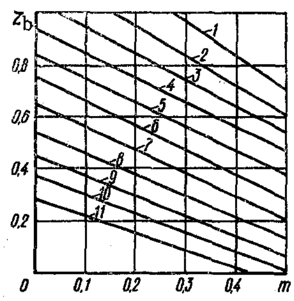

Fig. 3

Fig. 2. Relative height of liquid around the tube, for H = 0.03 and different values of the parameter m.

Fig. 3. Height of liquid at the boundary with the drip region of a tube (at $\psi = 150°$), for different values of parameters H and m: 1) H = 0.30; 2) 0.20; 3) 0.07; 4) 0.05; 5) 0.03; 6) 0.02; 7) 0.01; 8) 0.005; 8) 0.0025; 10) 0.001; 11) 0.0005.

On the basis of these assumptions one can determine the velocity of a liquid along the recess and the resulting rate of condensate flow along the groove:

$$G_\psi = \frac{\rho^2 h^4 \sin^3 \varphi}{12\mu} \sin \psi (z + m)^4, \tag{8}$$

where

$$z = \Delta/h; \quad m = a/h \operatorname{tg} \varphi,$$

as functions of the dimensionless immersion parameter z and the angular coordinate ψ.

The change in the rate of flow along the groove

$$dG_\psi = \frac{\rho^2 h^4 \sin^3 \varphi}{12\mu} [(z + m)^4 \cos \psi d\psi + 4 (z + m)^3 \sin \psi dz]$$

per element of length $R_0 d\psi$ is due to an efflux of the condensate, of the thickness is given by Eq. (6) and with the velocity given in Eq. (5), off the fin:

$$dG_\psi = \rho \bar{W}_y \delta R_0 d\psi = \frac{0.94 h^{1/2} \cos^{1/4} \varphi \sigma^{1/4} \lambda^{3/4} (t_s - t_W)^{3/4} R_0}{(1 + \operatorname{tg} \varphi)^{1/4} b^{1/4} \mu^{3/4} r^{3/4}} (1 - z)^{1/2} d\psi.$$

From the last two equations we obtain a differential equation which describes the height of liquid in the channel as a function of angle ψ:

$$\frac{dz}{d\psi} = H \frac{(1 - z)^{1/2}}{(z + m)^3 \sin \psi} - \frac{z + m}{4 \operatorname{tg} \psi}, \tag{9}$$

where

$$H = \frac{2{,}8 \sigma^{1/4} \mu^{3/4} \lambda^{3/4} R_0 (t_s - t_W)^{3/4}}{\sin^3 \varphi (1 + \operatorname{tg} \varphi)^{1/4} \cos^{1/4} \varphi \rho^{7/4} h^{7/2} b^{1/4} r^{3/4}}.$$

It appears from Eq. (9) that the height of liquid as a function of angle ψ is determined by two dimensionless parameters m and H. Parameter H represents the physical properties of the liquid, the mean temperature excess, and the surface geometry. Parameter m characterizes the fin geometry.

By virtue of symmetry with respect to the vertical axis of the tube in this problem,

$$\text{at} \quad \psi = 0 \qquad \frac{dz}{d\psi} = 0.$$

Moreover, from (9) we find the equation of the initial liquid height $(z_0 + m) = 4H(1 - z_0)^{1/2}$ where $z_0 = z|_{\psi=0}$.

Having found the the real roots of this algebraic equation, we obtain, after generalization, the boundary conditions in explicit form:

$$\text{at} \quad \psi = 0 \qquad z_0 = 1.01 H^{0.19} m^{0.34}. \tag{10}$$

The differential equation (9), with the boundary condition (10), was integrated numerically using the Euler method with certain refinements on the Mir computer.

The graph shown in Fig. 2 illustrates the profile of liquid height around the tube, for H = 0.03 and a series of m values from 0 to 0.5.

As follows from simple physical considerations, an increase of the parameter m, i.e., a widening of the groove between fins will result in a reduced height of liquid. Along the first portion of the path there takes place even some reduction of the film thickness, which is caused by the accelerating component of gravity. Along the lower portion of the tube, beginning at $\psi = 150°$, the height of liquid increases rapidly as the liquid decelerates and separates from the tube surface. Hydrodynamic studies [1, 5] concerning the flow of liquid films along horizontal tubes have also shown that the so-called drip region of the tube, where the liquid accumulates and separates from the surface in the form of single drops, constitutes a 60° zone.

In terms of the ψ-coordinate, this zone ($\psi = 150°$) will be considered the end of the region where vapor ceases to condense on a horizontal tube.

The differential equation (9) was solved numerically for the boundary with the drip region of a tube, and the results are plotted in the form $z_b = f(H, m)$ in Fig. 3. It can be seen here that the values of the parameters which define the conditions of vapor condensation on the tube, m from 0 to 0.5 and H from $0.5 \cdot 10^{-3}$ to $300 \cdot 10^{-3}$, encompass the entire range of liquid heights from zero to unity.

The calculated data shown in Fig. 3 can be put, within a ±2% accuracy, into a general equation:

$$z_b = 1.6H^{0.2}(1 - 0.35H^{-0.3}m). \tag{11}$$

Equation (11) or Figure 3 can be used directly for calculating the heat transfer during film condensation of vapor on a finned horizontal tube when We ≥ 10.

We will also consider now the problem concerning the mean (over the fin height) temperature drop $t_s - t_w$, with reference to which the basic parameter of the condensation process H must be calculated.

During tests one usually measures the temperature of a tube wall, which under best circumstances is the temperature at the root of a fin. According to the model of the process on which this analysis is based, the principal condensation occurs at the lateral surface of a fin, where the temperature differs from the temperature at the root.

In order to determine the law of temperature distribution over the height of a fin surface, one must solve the equation of heat conduction for the fin:

$$\frac{d^2\theta}{d\xi^2} = n\theta^{3/4}\xi^{-1/4}, \tag{12}$$

where

$$\theta = \frac{t_s - t_w}{t_s - t_0}; \qquad \xi = \frac{x\cos\varphi}{h};$$

$$n = \frac{1.4\rho^{1/4} r^{1/4} \sigma^{1/4} \lambda^{3/4} h^{3/2}}{\mu^{1/4} b^{1/4} \lambda_p (1 + \mathrm{tg}\,\varphi)(2b + h\sin\varphi)(t_s - t_0)^{1/4}}.$$

It is assumed here that the fin thickness remains constant and equal to the thickness at h/2, and that no temperature drop occurs across the fin thickness. The heat-transfer coefficient along the fin is determined from Eq. (6), which describes how the film thickness varies along the unimmersed portion of a fin according to the condition that

$$\alpha = \lambda/\delta.$$

If the solution to the equation of heat conduction (12)

$$\theta = [1 - 0.07n^{3/4}(1 - \xi)^{\xi}]^{9}, \tag{13}$$

obtained in [2] for the boundary conditions

$$\text{when } \xi = 0 \quad \frac{d\theta}{d\xi} = 0; \text{ and when } \xi = 1 \quad \theta = 1,$$

is integrated with respect to ξ from 0 to 1, then we will find the desired mean-integral value of the temperature drop along a fin as, approximately,

TABLE 1. Geometrical Characteristics of Finned Tubes

Number	Material	Tube radius at the fin roots R_0, mm	Fin height h, mm	One half of fin separation, a mm (at the base)	Half of thickness at top of fin b, mm	Angle of fin taper, φ deg	Parameter, m	We number for water	We number for Freon-113
1	Brass	9	0.92	0.07	0.07	16	0.27	70	14
2	Brass	9	0.92	0.07	0.07	28.5	0.14	60	12
3	Brass	9	1.32	0.07	0.07	11.5	0.27	50	9
4	Copper	8.5	2.05	0.10	0.32	16.5	0.17	7	1.5

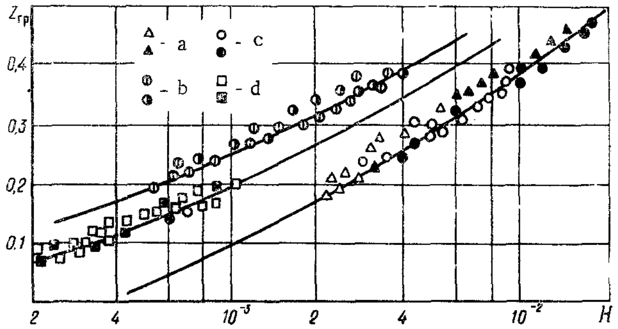

Fig. 4. Test data for water and Freon-113, generalized into the relation $z_b = f(H, m)$ (blank symbols for water, black symbols for Freon-113): a) tube No. 1, b) tube No. 2; c) tube No. 3; d) tube No. 4.

$$\tilde{\theta} = 0.38 + 0.62^{n+1} - 0.012n, \quad (14)$$

or, converting to dimensional quantities,

$$t_s \tilde{-} t_W = (0.38 + 0.62^{n+1} - 0.012n)(t_s - t_0). \quad (15)$$

Finally, we write down the mean heat-transfer coefficient for a finned horizontal tube, this coefficient being the basic characteristic of the vapor film condensation process:

$$\tilde{\alpha} = \frac{G_\psi r}{F_s (t_s - t_0)}, \quad (16)$$

where the rate of condensate flow G_ψ along half a groove is determined by Eq. (8) taking into account that $\sin\psi = 0.5$ at $\psi = 150°$ and where the working surface area F_S in the condensation process can be determined as follows:

$$F_s = \left(a + b + \frac{h}{\cos\varphi}\right)\pi R_0. \quad (17)$$

In order to verify the analytical calculations, an experimental apparatus was built and specimens of finned tubes were prepared for the purpose of heat-transfer measurements during film condensation of water and Freon-113 vapors on these tubes.

The design of the apparatus and the experimental procedure have been described in detail in [4].

The essential geometrical dimensions of four finned tubes used for these tests are given in Table 1. The first three tube specimens with fin heights 0.92 and 1.32 mm were prepared specially for this experiment. The fourth tube with a fin height h = 2.05 mm is a standard item produced in factories for refrigerator condensers. Its dimensions do not precisely match the requirements of our process model with We = 10, inasmuch as in this case We = 7 with water and We = 1.5 with Freon-113.

The tests were performed at a vapor pressure of 1.1 atm abs. and with temperature drops (referred to the temperature of the tube wall) in the range from 2 to 20°C.

We will briefly note that, in terms of the mean heat-transfer coefficient, the data for water and for Freon-113 on tube No. 4 (standard) agree with the results obtained in [4] for smooth tubes (i.e., no enhancement of the condensation process was observed). The first three tubes have 1.5-2.0 times higher heat-transfer coefficients than a smooth tube. According to the test data on water and Freon-113 vapor condensation, the heat-transfer coefficients of these substances – by virtue of their different physical properties (especially the heat of evaporation) – differ by a factor of 10-15.

The experimental data were evaluated according to the analytically derived relation $z_b = f(H, m)$. The results of this study are shown in Fig. 4, the test points for each tube specimen with water and with Freon-113 plotted on the same curve. Since the geometrical parameter m has the same value 0.27 for tubes No. 1 and No. 3, their respective test data overlap.

The three curves on the diagram correspond to the theoretical relation (17) plotted here for three values of parameter m: 0.14, 0.17, and 0.27 characterizing the geometry of the four tested tube specimens. It follows from this diagram that tested and calculated values agree within ±5%.

NOTATION

ρ	is the density of condensing liquid;
μ	is the dynamic viscosity;
σ	is the coefficient of surface tension;
λ	is the thermal conductivity of liquid;
λ_p	is the thermal conductivity of fin wall;
r	is the heat of evaporation of liquid;
t_S	is the vapor saturation temperature;

t_W	is the temperature of fin wall;
t_0	is the temperature of fin root;
$t_S - t_W$	is the mean (over the fin height) temperature excess;
h	is the fin height;
a	is the one half the fin separation (at the base);
b	is the one half the fin thickness at the top;
φ	is the angle of fin taper;
R_0	is the tube radius at the fin roots;
R_T	is the curvature radius of the liquid film on top of fin;
δ, Δ	are the condensate film thickness, on the fin and in the channel, respectively;
p	is the vapor pressure on the liquid film;
x	is the coordinate along the fin surface, where condensate flows from top to base;
y	is the coordinate normal to the lateral surface of a fin;
W	is the velocity of condensate flowing from overhang to drain;
U	is the velocity of liquid film flowing along the channel between fins;
ψ	is the azimuthal angle, measured from top point of a horizontal tube;
G_x, G_ψ	are the flow rates along coordinates x and ψ respectively;
α	is the heat-transfer coefficient.

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